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Application of dipole sum rules to transfer reaction strengths

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Abstract. Equations illustrating the application of dipole sum rules by relating the reaction strengths from single-particle transfer (stripping as well as pick up reactions) to the magnetic dipole moment of the target state (derived earlier) have been rewritten in a more symmetrical and user friendly form. The purpose of the present work is not to calculate the magnetic moment but to provide six different ways —from stripping and pick up reactions as well as from their combination, to study the discrepancies in the measurement of reaction strengths through their relationships with the magnetic moment.

PACS. 25.40.-h Nucleon induced reactions – 21.30.-x Nuclear forces – 11.55.Hx Sum rules

1 Introduction

Multipole sum rule methods, within the framework of shell model, were designed to provide useful information about nuclear structure without resorting to large-scale spectroscopic calculations. A large number of applications of the monopole sum rules (non-energy-weighted as well as linear energy-weighted) have been reported [1–11] but higher-order sum rules have been applied sparingly [12–14].

The present article focuses itself on the relationships [13] (obtained through the use of dipole sum rules) between single-particle transfer strengths and a measurable physical quantity, that is, the magnetic dipole moment of the target state. The purpose is not to calculate the magnetic moments, but to check the discrepancies in measured strengths through these relationships.

The present work is an advancement of our previous work [13,14] where only stripping reactions were utilized to exploit such mutual relationships. Now the pick up reactions as well as the combination of stripping and pick up reactions have also been used, which provides six options, in all, to test the quality of measured strengths towards building up the known magnetic moments. It should be emphasized that the dipole sum rule has the advantage of being sensitive to the distribution of strengths among various J-states of the final nucleus whereas the monopole sum rule showed sensitivity only to the total strength for all J-states. The lack of agreement between the measured magnetic moment (known so precisely), and our extracted values is a reflection, in part, on the discrepancies in the measured stripping/pick up reaction strengths.

This work is also an improvement on our previous contribution [13] in another sense. The relationships between magnetic moment of the target state and single-particle transfer reaction strengths have now been presented in a more elegant form which makes it more convenient to handle them.

2 Algebraic apparatus

The equations, relating the target state magnetic moment to the spectroscopic factors of the states of residual nuclei, obtained via single-particle transfer reactions on the given target, were derived in one of our earlier works [13]. However, to make this article reasonably self-contained, we are giving below the necessary background and the steps involved in the derivation of these equations. Various terms in the equations are also being recast to make them look more symmetrical.

2.1 Notation

We denote the target state involved in a typical transfer reaction by $\mid n\Gamma_0 x_0\rangle$, the particle transfer orbit by ρ and the final state of the residual nucleus by $\mid n \pm 1, \Gamma x\rangle$. Following French [2], the Greek letters here represent both the spin (angular momentum) and isospin, so that $\Gamma_0 \equiv J_0 T_0$; $\rho \equiv j1/2$ and $\Gamma \equiv JT$. The label n gives the number of active nucleons in the target state, while x_0 and x stand for all the non–angular-momentum quantum numbers required to uniquely define the target state

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and the final state, respectively. Any algebraic factors involving Greek letters in the expressions or equations occuring in the text would actually represent a product of two factors (one involving spin and the other involving isospin). Thus, $(-1)^{\Gamma} = (-1)^{J+T}$; $U(\Gamma_0\rho\Gamma_0\rho;\Gamma\Lambda) = U(J_0jJ_0j;Jk)U(T_0 1/2 T_0 1/2;Tt)$ etc.
We also use the notation,

$$[\Gamma] \equiv [J][T] = (2J+1)(2T+1) \text{ etc.}$$

Angular momentum coupling will be represented by a "$\times$", *e.g.*, $(A^{\rho} \times B^{\rho})^{\Lambda}$ represents the coupling of an operator A^{ρ} (rank ρ) with an operator B^{ρ} (rank ρ) to form an operator of rank Λ by the usual angular momentum coupling rules.

2.2 Cartesian operators and unit tensors

In order to look after the angular momentum and antisymmetry requirements, we prefer to work in the second quantized tensorial formalism developed by French [2]. In this formalism, the nucleon creation and destruction operators for the shell model orbit ρ are represented by A^{ρ} and B^{ρ}, respectively. The second quantized representation of the symmetrical single-particle unit tensor (introduced initially by Racah) in terms of the fundamental creation and destruction operators is given by [2]

$$U^{\Lambda}_{\rho\rho'} \equiv [\Lambda]^{-1/2}(A^{\rho} \times B^{\rho'})^{\Lambda}\,, \tag{1}$$

such that, for a properly normalised one particle state $\mid \rho\rangle$, we have

$$\langle \rho \mid\mid U^{\Lambda}_{\rho\rho'} \mid\mid \rho'\rangle = 1\,. \tag{2}$$

Making use of the fact that the m-dependence of the matrix elements of any two tensor operators of the same rank is the same, we can write for any symmetrical one-body operator, O^{Λ}, which is diagonal in the quantum numbers ρ (the operators $\mathbf{j}$ and $\mathbf{t}$ with which we deal in this work are such operators),

$$\langle \rho \mid O^{\Lambda} \mid \rho\rangle = C\langle \rho \mid U^{\Lambda}_{\rho\rho} \mid \rho\rangle\,. \tag{3}$$

The constant of proportionality, C, being independent of m, is just the ratio of the reduced matrix elements of the two tensors. Therefore, we can write the part of the operator O^{Λ} corresponding to orbit ρ in terms of the U^{Λ}'s as follows:

$$O^{\Lambda}(\rho) = \langle \rho \mid\mid O^{\Lambda} \mid\mid \rho\rangle U^{\Lambda}_{\rho\rho}\,. \tag{4}$$

It is well known that the reduced matrix element of the angular momentum operator $\mathbf{j}$ (a vector operator or a tensor of rank 1 in J-space) between single-particle states is given by

$$\langle j \mid\mid \mathbf{j} \mid\mid j\rangle = \sqrt{j(j+1)(2j+1)} = \sqrt{j(j+1)[j]}\,. \tag{5}$$

Noting that $\mathbf{j}$ is a scalar in the isospin space, we can extend this result to the JT-space and write

$$\langle \rho \mid\mid \mathbf{j} \mid\mid \rho\rangle = \sqrt{j(j+1)[\rho]}\,. \tag{6}$$

Hence the expression for $\mathbf{j}(\rho)$ in terms of the unit tensors, becomes

$$\mathbf{j}(\rho) = \sqrt{j(j+1)[\rho]}\; U^{10}_{\rho\rho}\,. \tag{7}$$

Proceeding on similar lines, we shall obtain

$$\begin{aligned} \mathbf{jt}(\rho) &= \sqrt{j(j+1)(1/2)(3/2)[\rho]}\; U^{11}_{\rho\rho} \\ &= \frac{\sqrt{3}}{2}\sqrt{j(j+1)[\rho]}\; U^{11}_{\rho\rho}\,, \end{aligned} \tag{8}$$

where $\mathbf{t}$ is the isospin operator (rank 1 in isospin space) and we have made use of the fact that the isospin of a single-nucleon state is 1/2.

2.3 Magnetic moment operator

It is customary to define g_j, a g-factor for a particle in a state with a definite value of j, such that the magnetic moment of the particle in that state is given by

$$\mu = g_j j\,. \tag{9}$$

We also have the well-known results that

$$\begin{aligned} g_j &= g_l - \tfrac{g_l - g_s}{2j} && \text{for } \; j = l + 1/2\,, \\ &= g_l + \tfrac{g_l - g_s}{2(j+1)} && \text{for } \; j = l - 1/2\,. \end{aligned} \tag{10}$$

It is easy to verify that the following single expression encompasses both of these results:

$$g_j = g_l - \frac{(-1)^{l+1/2-j}}{2l+1}(g_l - g_s)\,. \tag{11}$$

Noting that the orbital gyromagnetic ratios for a proton and a neutron are $1\mu_0$ and 0, respectively, and writing $g_p\mu_0$ and $g_n\mu_0$ for their respective spin gyromagnetic ratios, we have for a proton and a neutron, respectively, in the orbit $\rho(lj)$,

$$\begin{aligned} \frac{g_j(p)}{\mu_0} &= 1 - (1 - g_p)\frac{(-1)^{l+1/2-j}}{2l+1}\,, \\ \frac{g_j(n)}{\mu_0} &= g_n\frac{(-1)^{l+1/2-j}}{2l+1}\,, \end{aligned} \tag{12}$$

where μ_0 stands for the nuclear magneton.
We prefer to work in the JT-space where proton and neutron are just two forms of a nucleon, with isospin projection $t_z = -1/2$ and $+1/2$, respectively. Thus we may write for a nucleon in the orbit ρ,

$$\begin{aligned} \frac{g_j}{\mu_0} &= (1/2 - t_z)\left[1 - (1 - g_p)\frac{(-1)^{l+1/2-j}}{2l+1}\right] \\ &\quad + (1/2 + t_z)g_n\frac{(-1)^{l+1/2-j}}{2l+1}\,, \end{aligned} \tag{13}$$

which after a little rearrangement becomes

$$\frac{g_j}{\mu_0} = \frac{1}{2}\left[1 + (g_n + g_p - 1)\frac{(-1)^{l+1/2-j}}{2l+1}\right] - \left[1 - (g_n - g_p + 1)\frac{(-1)^{l+1/2-j}}{2l+1}\right] t_z \,. \qquad (14)$$

The magnetic moment of a nucleon in the orbit ρ is then given by

$$\frac{\mu}{\mu_0} = \frac{g_j j}{\mu_0} = \frac{1}{2}\left[1 + (g_n + g_p - 1)\frac{(-1)^{l+1/2-j}}{2l+1}\right] j - \left[1 - (g_n - g_p + 1)\frac{(-1)^{l+1/2-j}}{2l+1}\right] j t_z \,. \qquad (15)$$

By definition, the magnetic moment of a state is the expectation value of $(\mu_{\rm op})_z$ in that state with the maximum projection in J-space, that is, $m = j$. Therefore, we may write

$$\frac{\mu_{\rm op}(\rho)}{\mu_0} = \frac{1}{2}\left[1 + (g_n + g_p - 1)\frac{(-1)^{l+1/2-j}}{2l+1}\right] \mathbf{j}(\rho) - \left[1 - (g_n - g_p + 1)\frac{(-1)^{l+1/2-j}}{2l+1}\right] \mathbf{jt}(\rho) \,. \quad (16)$$

Making use of the expressions for the operators, $\mathbf{j}$ and $\mathbf{jt}$ from eqs. (7) and (8) and in view of the fact that each active nucleon contributes towards the nuclear magnetic moment, the magnetic moment operator for a nucleus, as far as its diagonal matrix elements are concerned, may be written as

$$\frac{\mu_{\rm op}}{\mu_0} = \frac{1}{2}\sum_{\rho}\left\{1 + (g_n + g_p - 1)\frac{(-1)^{l+1/2-j}}{2l+1}\right\} \times \{j(j+1)[\rho]\}^{1/2} U^{10}_{\rho\rho} - \frac{\sqrt{3}}{2}\sum_{\rho}\left\{1 - (g_n - g_p + 1)\frac{(-1)^{l+1/2-j}}{2l+1}\right\} \times \{j(j+1)[\rho]\}^{1/2} U^{11}_{\rho\rho} \,, \qquad (17)$$

where the summation runs over all the active orbits.

2.4 Magnetic moment and spectroscopic factors

With the form of the magnetic moment operator given in the previous subsection, what we need now, to evaluate the magnetic moment of a state, are essentially the matrix elements of the operators U^{10} and U^{11} in that state. The multipole sum rules for single-particle transfer reactions give us the expressions for these matrix elements (in the target state) in terms of the spectroscopic factors of the states of the final nucleus. The basic sum rule equations for single-particle stripping and pick up cases, respectively, are [2]

$$\langle n\Gamma_0 x_0 \mid\mid U^{\Lambda}_{\rho\rho} \mid\mid n\Gamma_0 x_0\rangle = [\Gamma_0\rho]^{1/2}\delta_{\Lambda 0} - [\Lambda]^{-1/2}\sum_{\Gamma}(-1)^{\Gamma_0+\rho-\Gamma}[\Gamma]^{1/2}U(\Gamma_0\rho\Gamma_0\rho;\Gamma\Lambda)S^{+}_{\Gamma} \,, \quad (18)$$

$$\langle n\Gamma_0 x_0 \mid\mid U^{\Lambda}_{\rho\rho} \mid\mid n\Gamma_0 x_0\rangle = (-1)^{\Lambda}[\Lambda]^{-1/2} \times \sum_{\Gamma}(-1)^{\Gamma_0+\rho-\Gamma}\frac{[\Gamma_0]}{[\Gamma]^{1/2}}U(\Gamma_0\rho\Gamma_0\rho;\Gamma\Lambda)S^{-}_{\Gamma} \,, \qquad (19)$$

where $S^{+}_{\Gamma} \equiv \sum_x S^{+}_{\Gamma x}(n\Gamma_0 x_0 + \rho \to n+1, \Gamma x)$ and $S^{-}_{\Gamma} \equiv \sum_x S^{-}_{\Gamma x}(n\Gamma_0 x_0 - \rho \to n-1, \Gamma x)$ are the summed spectroscopic factors for all states of the final nucleus having a particular value of Γ in the case of stripping (superscript '+') and pick up (superscript '−') reactions, respectively. The summations on the right-hand sides of these equations run over all possible values of J and T ($\Gamma \equiv JT$) and for a given target isospin T_0, there are only two possible values of the final state isospin T, these being $T_< \equiv (T_0 - 1/2)$ and $T_> \equiv (T_0 + 1/2)$.

In certain experimental situations, states belonging to both these isospins may not be observed in a single reaction. In that case, the sum rule equations can be inverted with regard to the isospin variables [12]. Making use of the analytical values of the Racah coefficients $U(T_0 1/2 T_0 1/2; T0)$ and $U(T_0 1/2 T_0 1/2; T1)$, now occuring on the left hand side, we can obtain matrix elements of the linear combinations of the isoscalar and isovector unit tensors, as

$$\langle nJ_0T_0x_0 \mid\mid U^{k0}_{\rho\rho} + \sqrt{3}\;\alpha(T_0, T^{+})U^{k1}_{\rho\rho} \mid\mid nJ_0T_0x_0\rangle = [\rho\Gamma_0]^{1/2}\delta_{k0} - \sqrt{2}\frac{[T_0]^{1/2}}{[k]^{1/2}} \times \sum_{J}(-1)^{J_0+j-J}[J]^{1/2}U(J_0jJ_0j;Jk)S^{+}_{JT} \qquad (20)$$

and

$$\langle nJ_0T_0x_0 \mid\mid U^{k0}_{\rho\rho} - \sqrt{3}\;\alpha(T_0, T^{-})U^{k1}_{\rho\rho} \mid\mid nJ_0T_0x_0\rangle = \sqrt{2}(-1)^{k}\frac{[\Gamma_0]}{[T^{-}]}\frac{[T_0]^{1/2}}{[k]^{1/2}} \times \sum_{J}(-1)^{J_0+j-J}[J]^{-1/2}U(J_0jJ_0j;Jk)S^{-}_{JT} \,, \qquad (21)$$

where

$$\alpha(T_0, T) = -\frac{(T_0+1)}{\sqrt{T_0(T_0+1)}} \quad \text{for} \quad T = T_< \,, \qquad = \frac{T_0}{\sqrt{T_0(T_0+1)}} \quad \text{for} \quad T = T_> \,. \qquad (22)$$

Equations (20) and (21) can be solved to get $\langle\mid\mid U^{k0}_{\rho\rho} \mid\mid\rangle$ and $\langle\mid\mid U^{k1}_{\rho\rho} \mid\mid\rangle$ in terms of a combination of spectroscopic factors for stripping and pick up reactions performed on the same target state $\mid n\Gamma_0 x_0\rangle$. The sum rule equations (18), (19), (20) and (21) may be combined with the expression for magnetic moment operator (17) to give us the relationships between magnetic moment of a target state and the spectroscopic factors for transfer reactions involving i) pick up of a particle alone, ii) stripping alone and iii) a combination of pick up and stripping. The relationships obtained by this procedure are

$$\frac{\mu}{\mu_0} = -\frac{1}{4(J_0+1)(T_0+1)} \sum_{\rho J} F(\rho, J) \times \Big[\{(T_0+1)X_1(\rho) + T_0 X_2(\rho)\} S^-_{JT_>} + (T_0+1)\{X_1(\rho) - X_2(\rho)\} S^-_{JT_<} \Big] , \tag{23}$$

$$\frac{\mu}{\mu_0} = -\frac{1}{2(J_0+1)(2T_0+1)} \sum_{\rho J} F(\rho, J) \left(\frac{2J+1}{2J_0+1}\right) \times \Big[\{(T_0+1)X_1(\rho) - T_0 X_2(\rho)\} S^+_{JT_>} + T_0\{X_1(\rho) + X_2(\rho)\} S^+_{JT_<} \Big] , \tag{24}$$

and

$$\frac{\mu}{\mu_0} = -\frac{1}{2(J_0+1)[f(T^+) + f(T^-)]} \sum_{\rho J} F(\rho, J) \times \Big[\{f(T^-)X_1(\rho) - T_0 X_2(\rho)\} \left(\frac{2J+1}{2J_0+1}\right) S^+_{JT^+} + \{f(T^+)X_1(\rho) + T_0 X_2(\rho)\} \left(\frac{2T_0+1}{2T^-+1}\right) S^-_{JT^-} \Big] , \tag{25}$$

where

$$F(\rho, J) = J(J+1) - J_0(J_0+1) - j(j+1); \tag{26}$$

$$f(T) = T(T+1) - T_0(T_0+1) - \frac{3}{4}; \tag{27}$$

$$X_1(\rho) = 1 + (g_n + g_p - 1)\frac{(-1)^{l+1/2-j}}{2l+1} = 1 + 0.759\frac{(-1)^{l+1/2-j}}{2l+1}; \tag{28}$$

$$X_2(\rho) = 1 - (g_n - g_p + 1)\frac{(-1)^{l+1/2-j}}{2l+1} = 1 + 8.411\frac{(-1)^{l+1/2-j}}{2l+1} . \tag{29}$$

In the last two equations, we have used the values of g_p ($= 5.585$) and $g_n (= -3.826)$ for the spin gyromagnetic ratios of a proton and a neutron, respectively.

Pick up of a neutron from a target can, in general, populate states of the residual nucleus having isospin $T_< = T_0 - 1/2$ as well as $T_> = T_0 + 1/2$. If sufficient strengths for both these isospin bands are seen in a pick up reaction, then eq. (23) can give us the value of μ, the magnetic moment of the target state. Similarly, in a proton stripping experiment, in general, both $T_<$- and $T_>$-states of residual nucleus can be populated and if sufficient strength is observed for both these isospin values, eq. (24) alone is sufficient to estimate μ.

However, for states with both values of isospin, $T_<$ and $T_>$, if sufficient strength is not seen simultaneously in a stripping or in a pick up reaction alone, then eq. (25) is required which gives μ in terms of spectroscopic factors of states having sufficient strength belonging to a particular value of T ($T_<$ or $T_>$) both for stripping as well as pick up situations. In this situation there are four possibilities; the combination of i) $T_<$- states of stripping with $T_<$- states of pick up, ii) $T_<$- states of stripping with $T_>$- states of pick up, iii) $T_>$- states of stripping with $T_<$- states of pick up and iv) $T_>$- states of stripping with $T_>$- states of pick up. Any one of these combinations may be used in the case of eq. (25). Thus, if in both stripping and pick up reactions, adequate data are available for a given target nucleus and in both cases, $T_<$- states and $T_>$- states are populated with sufficient strength, then with the help of eqs. (23), (24) and (25), we can estimate the value of μ in six different ways.

As can be seen from the eqs. (23), (24) and (25), the value of μ is sensitive to the distribution of strength among various states of residual nucleus. The contribution of states having different J values can have different signs depending on the factor $[J(J+1) - J_0(J_0+1) - j(j+1)]$. Thus, it is important that apart from accurate strength measurements, the experimental data should also give accurate assignments of J, for various states, to get meaningful results from the above equations.

3 Calculations and results

We have collected from the literature [15–40], experimental data for proton stripping and neutron pick up reactions on the targets ^{17}O, ^{23}Na, ^{25}Mg, ^{27}Al, ^{29}Si, ^{31}P, ^{33}S, ^{35}Cl, ^{37}Cl, ^{39}K, ^{41}Ca, ^{43}Ca and ^{45}Sc. We have chosen the proton stripping and neutron pick up cases because it is well known that in such reactions, both $T_<$-states and $T_>$-states of the residual nucleus, can in general be populated. The selection of target nuclei has also been guided by the availability of data for both $T_<$-states and $T_>$-states, as far as possible. Assuming the simplest possible pure shell model configurations for the target states, we have calculated, using the non-energy-weighted sum rules [1], the strengths expected for particle transfer taking place to different shell model orbits. These are compared with the experimentally observed strengths given in tables 1 and 2.

As mentioned above, in most of the chosen cases, both $T_<$-states and $T_>$-states of residual nuclei are seen in the pick up as well as in stripping experiments. With the help of eqs. (23) and (24) we have calculated the contribution towards magnetic moment from nucleons populating various orbits of the target state using data from pick up and stripping reactions, separately. The results are shown in table 3 under the headings, Calc. 1 and Calc. 2, respectively. We find that the values obtained from pick up reaction data (Calc. 1), in almost all cases, are in better agreement with the experimentally measured values of magnetic moments (also given in table 3) than those obtained from stripping reaction data alone (Calc. 2). We, therefore, tend to believe that the strengths are better measured in pick up reactions than in the stripping reactions chosen by us. Also, in a particular reaction, the total

Table 1. Strengths for proton stripping reactions.

Target Nucleus	Ref. for expt. data	Assumed configuration	Transfer orbit	Strength $T_<$-states observed	$T_<$-states expected	$T_>$-states observed	$T_>$-states expected
^{17}O	16	$(^{16}\text{O})1d^1_{5/2}$	$1d_{5/2}$	3.21	3.50	2.69	2.50
^{23}Na	18	$(^{16}\text{O})1d^7_{5/2}$	$1d_{5/2}$	1.94	2.00	0.67	1.00
			$2s_{1/2}$	0.27	1.00	0.39	1.00
^{25}Mg	20	$(^{16}\text{O})1d^9_{5/2}$	$1d_{5/2}$	1.41	1.50	0.43	0.50
			$2s_{1/2}$	0.68	1.00	0.31	1.00
^{27}Al	22	$(^{16}\text{O})1d^{11}_{5/2}$	$1d_{5/2}$	0.51	1.00	—	—
^{29}Si	24	$(^{16}\text{O})1d^{12}_{5/2}2s^1_{1/2}$	$2s_{1/2}$	0.85	1.50	0.34	0.50
^{31}P	26	$(^{16}\text{O})1d^{12}_{5/2}2s^3_{1/2}$	$2s_{1/2}$	0.87	1.00	0.30	0.0
^{33}S	28	$(^{16}\text{O})1d^{12}_{5/2}2s^4_{1/2}1d^1_{3/2}$	$1d_{3/2}$	2.11	2.50	1.22	1.50
			$1f_{7/2}$	2.69	4.00	—	4.00
^{35}Cl	30	$(^{16}\text{O})1d^{12}_{5/2}2s^4_{1/2}1d^3_{3/2}$	$1d_{3/2}$	2.07	2.00	0.48	1.00
^{37}Cl	30	$(^{16}\text{O})1d^{12}_{5/2}2s^4_{1/2}1d^5_{3/2}$	$1d_{3/2}$	3.13	3.00	—	—
^{39}K	33	$(^{16}\text{O})1d^{12}_{5/2}2s^4_{1/2}1d^7_{3/2}$	$1d_{3/2}$	0.60	1.00	—	—
^{41}Ca	35	$(^{40}\text{Ca})1f^1_{7/2}$	$1f_{7/2}$	3.13	4.50	2.31	3.50
^{43}Ca	37	$(^{40}\text{Ca})1f^3_{7/2}$	$1f_{7/2}$	4.98	6.75	0.11	1.25
^{45}Sc	39	$(^{40}\text{Ca})1f^5_{7/2}$	$1f_{7/2}$	4.21	6.00	0.28	1.00

observed strength for the $T_>$-states usually falls shorter of the theoretically expected value by a larger margin than that in the case of $T_<$-states. Hence, we tend to come to the general conclusion that in the available data, the strengths of $T_>$-states observed in stripping reactions, are less reliable than the others.

As discussed in the previous section, eq. (25) provides us with four different ways of combining the pick up and stripping reaction data. However, in view of the above remarks, we have tried only two combinations, namely, $T_<$-states and $T_>$-states from pick up reactions combined, in turn, with $T_<$-states from stripping reactions. The results so obtained are included in table 3 under the headings, Calc. 3 and Calc. 4, respectively.

Ideally speaking, all the four calculations should yield identical results, but keeping in view the practical situation that the DWBA used for extracting strengths from experimental observations is not an exact theory, we do expect some variations in the results of our four calculations. Large variations observed in some cases are, however, a cause of concern. These discrepancies could result from any of the following reasons:

i) error in strength measurments
ii) imprecise J/T assignments of nuclear states.

A close look at table 3 shows that the results of our calculations show good mutual consistency in the case of ^{17}O, ^{23}Na, ^{25}Mg, ^{31}P, ^{33}S and ^{35}Cl nuclei though these do not always agree with the known values of magnetic moments of these nuclei because the contribution of nucleons in all the active orbits may not be known from the available data. We can, therefore, say that in both the stripping and pick up reactions reported on these nuclei [16–21, 26–31], the strength measurements for transfer to the main active orbit in each case, are reasonably accurate.

In the case of ^{27}Al, ^{41}Ca and ^{45}Sc, we find that wherever stripping strengths are involved (Calc. 2, Calc. 3 and Calc. 4), the results are numerically much smaller than those of Calc. 1 where only pick up strengths have been used. The reason for this appears to be the fact that the total expected value of strength for stripping of a particle into the relevant orbit is not seen in the experiments performed on each of these nuclei. This is substantiated by a perusal of the figures given in table 1 where the total expected strengths are compared with the experimentally

Table 2. Strengths for neutron pick up reactions.

Target Nucleus	Ref. for expt. data	Assumed configuration	Transfer orbit	Strength $T_<$-states observed	Strength $T_<$-states expected	Strength $T_>$-states observed	Strength $T_>$-states expected
^{17}O	17	$(^{16}\mathrm{O})1d^1_{5/2}$	$1d_{5/2}$	1.00	1.00	0.00	0.00
^{23}Na	19	$(^{16}\mathrm{O})1d^7_{5/2}$	$1p_{1/2}$	0.70	1.00	0.74	1.00
			$1d_{5/2}$	1.72	2.50	0.88	1.50
^{25}Mg	21	$(^{16}\mathrm{O})1d^9_{5/2}$	$1d_{5/2}$	2.79	3.00	0.94	2.00
^{27}Al	23	$(^{16}\mathrm{O})1d^{11}_{5/2}$	$1d_{5/2}$	2.08	3.50	1.59	2.50
^{29}Si	25	$(^{16}\mathrm{O})1d^{12}_{5/2}2s^1_{1/2}$	$2s_{1/2}$	0.59	1.00	0.08	0.00
^{31}P	27	$(^{16}\mathrm{O})1d^{12}_{5/2}2s^3_{1/2}$	$1d_{5/2}$	2.70	3.00	0.92	3.00
			$2s_{1/2}$	1.06	1.50	0.37	0.50
^{33}S	29	$(^{16}\mathrm{O})1d^{12}_{5/2}2s^4_{1/2}1d^1_{3/2}$	$1d_{3/2}$	1.23	1.00	—	—
			$1f_{7/2}$	0.10	0.00	—	—
^{35}Cl	31	$(^{16}\mathrm{O})1d^{12}_{5/2}2s^4_{1/2}1d^3_{3/2}$	$1d_{3/2}$	2.59	1.50	0.91	0.50
^{37}Cl	33	$(^{16}\mathrm{O})1d^{12}_{5/2}2s^4_{1/2}1d^5_{3/2}$	$1d_{3/2}$	3.79	3.75	0.14	0.25
^{39}K	34	$(^{16}\mathrm{O})1d^{12}_{5/2}2s^4_{1/2}1d^7_{3/2}$	$2s_{1/2}$	0.30	1.00	0.03	1.00
			$1d_{3/2}$	1.91	2.50	0.70	1.50
^{41}Ca	36	$(^{40}\mathrm{Ca})1f^1_{7/2}$	$1f_{7/2}$	0.81	1.00	—	—
^{43}Ca	38	$(^{40}\mathrm{Ca})1f^3_{7/2}$	$1f_{7/2}$	1.88	3.00	—	—
^{45}Sc	40	$(^{40}\mathrm{Ca})1f^5_{7/2}$	$1f_{7/2}$	3.95	3.75	0.22	0.25

observed values. For instance, the observed strength for $T_<$-states in the case of proton stripping into the $1d_{5/2}$ orbit of ^{27}Al target, is about 50% of the expected value.

For ^{29}Si and ^{43}Ca targets, in the case of neutron pick up from $2s_{1/2}$ and $1f_{7/2}$ orbits, respectively, we find from table 2 that the measured strengths for $T_<$-states are about 60% of the expected values in each case, while the $T_>$-states are thoeretically untenable in both these cases. This perhaps results in the relatively smaller magnitudes of the contributions towards their respective magnetic moments calculated on the basis of pick up strengths alone (Calc. 1) as compared to the results based wholly or partly on stripping strengths (Calc. 2, Calc. 3 and Calc. 4 of table 3).

In the case of ^{37}Cl and ^{39}K ($1d_{3/2}$ transfer), we find from table 3, that the results of various calculations show rather large deviations from one another. Even the sign of the magnetic moment contributions obtained from Calc. 4 happens to be opposite to that of the results given by other calculations. A plausible explanation for this disagreement is that the observed strengths for $T_>$-states in the case of pick up reactions on each of these nuclei, happen to be only about 50% of their expected values. It has been verified by us that in the event of these strengths being fully observed (by arbitrarily doubling the observed strengths), the sign of the results of Calc. 4 would also match with those of the other results.

The results given above reinforce our earlier remarks that the values of the magnetic moment contributions extracted with the help of dipole sum rules are very sensitive to the discrepancies in strength measurements, and perhaps also to the assignments of J-values of various states.

4 Concluding remarks

In one of our earlier works [13], we had derived explicit relationships between magnetic dipole moment of a target state and the strengths of states of residual nuclei obtained via single-particle stripping as well as pick up reactions performed on this target. We have now put these relationships in a form which appears to be much more symmetrical and more convenient to handle.

Table 3. Calculated orbitwise contribution towards magnetic moment.

Target nucleus	$\frac{\mu_{\text{expt}}}{\mu_0}$	Calculated contribution towards magnetic moment				
		orbit	Calc. 1	Calc. 2	Calc. 3	Calc. 4
^{17}O	−1.894	$1d_{5/2}$	−1.78	−1.53	−1.59	−1.64
^{23}Na	2.217	$1p_{1/2}$	−0.10	—	—	—
		$1d_{5/2}$	2.30	2.84	2.93	2.77
		$2s_{1/2}$	—	−0.40	—	—
^{25}Mg	−0.855	$1d_{5/2}$	−0.67	−0.54	−0.33	−0.28
		$2s_{1/2}$	—	−0.17	—	—
^{27}Al	3.641	$1d_{5/2}$	3.75	2.33	2.33	2.69
^{29}Si	−0.555	$2s_{1/2}$	−0.71	−1.14	−1.05	−0.92
^{31}P	1.132	$1d_{5/2}$	−0.52	—	—	—
		$2s_{1/2}$	2.01	2.24	2.21	2.32
^{33}S	0.643	$1d_{3/2}$	1.21	1.01	1.24	1.13
		$1f_{7/2}$	—	—	−0.55	—
^{35}Cl	0.822	$1d_{3/2}$	0.57	0.34	0.49	0.77
^{37}Cl	0.684	$1d_{3/2}$	0.46	0.12	0.71	−2.12
^{39}K	0.391	$2s_{1/2}$	−0.04	—	—	—
		$1d_{3/2}$	0.43	0.07	0.67	−0.16
^{41}Ca	−1.595	$1f_{7/2}$	−1.55	−0.74	−0.51	−0.94
^{43}Ca	−1.317	$1f_{7/2}$	−1.31	−2.78	−2.86	−3.41
^{45}Sc	4.756	$1f_{7/2}$	4.61	2.96	3.09	2.56

Calc. 1: Uses eq. (23) with strengths of $T_<$- and $T_>$-states from a pick up reaction.
Calc. 2: Uses eq. (24) with strengths of $T_<$- and $T_>$-states from a stripping reaction.
Calc. 3: Uses eq. (25) with strengths of $T_<$-states obtained from pick up and those of $T_<$-states obtained from a stripping reaction.
Calc. 4: Uses eq. (25) with strengths of $T_>$-states obtained from pick up and those of $T_<$-states obtained from a stripping reaction.

We want to emphasize that our aim is not to give a prescription for the calculation of the magnetic moment of a nucleus (more accurate methods are available for this purpose). We are using this physical quantity merely as a representative to illustrate the effectiveness of dipole sum rules in pointing out the discrepancies in strength measurements. As mentioned earlier, our eqs. (23), (24) and (25) provide us with six possible ways to easily calculate the contribution towards magnetic moment from nucleons in a particular orbit in the target state (involved in particle transfer). If the measurement of strengths is accurate enough, results of all the six different calculations should show small variations from one another. Relatively large deviations seen in the results of our different calculations with respect to one another as well as with respect to the known value of magnetic moment, point towards the fact that we are still far away from accurate measurements of spectroscopic strengths.

We may reiterate that the dipole sum rules, being sensitive to the distribution of strength among various J-states, provide a more stringent check on the accuracy of strength measurements as compared to the monopole sum rules.

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